The Construction of the Concept of Binary Search Algorithm

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Abstract. The general goal of this paper is to describe how Piaget's theory-*Genetic Epistemology*-can contribute to computer science education research. Using, as an example, students' understanding of the binary search algorithm, this paper illustrates how theoretical principles are applied to identify some obstacles that students face while solving an instance of that algorithm and to help students to surmount these obstacles. The methodology of research consists in conducting students interviews in which they are encouraged to express their solutions in natural language. The specific goal of the paper is to show how Piaget's ideas are used as guidelines in the design of the interviews and in the analysis of the gathered information. Selected excerpts from students' interviews are included.

1 Introduction

The study presented in this paper is based on Jean Piaget's theory *genetic epistemology*, especially on Piaget's and collaborators' works regarding the psychological evolution of mathematical concepts and theories[12, 13]. Their investigations are supported by an extensive amount of empirical work. The importance of Piaget's work for computer science education lies in the fact that *the methods* that the subject employs to solve instances of problems play a central role in the explanations about the construction of knowledge. In this sense, the theory gives satisfactory support to the study of the psychogenetic evolution of algorithms, from which invaluable lessons can be taken when it comes to preparing instructional proposals.

The main theoretical principle in which our research is based is: the source of the instrumental knowledge is the interaction between the subject and the structure over which the algorithm is applied. This interaction is governed by the *general law of cognition*, that explains how instrumental knowledge can be transformed into conceptual knowledge. In Piaget's theory, the principal instrument of the whole process is called *reflective abstraction*[8, 11, 13].

In this paper we present an example of application of the above principle to the construction of knowledge about the binary search algorithm. The goal of the study is to gather information about the process of transforming the instrumental knowledge constructed by the students while solving an instance of the problem of searching an element in an ordered list into conceptual knowledge about the employed algorithm. The aim of the paper is to show how Piaget's explanations about the instruments and mechanisms involved in the construction of knowledge are used as guidelines of the study.

In this sense our work intends to contribute to computer science education research, accordingly to [2]. There, the authors give the following definition of the expression "computer science education":

The academic discipline computer science education consists in focusing research on the application of principles from educational-related disciplines -pedagogy, epistemology, curriculum studies and psychology- to the teaching and learning of the scientific discipline computer science as a school subject.

They add that the strong connection with educational-related disciplines constitutes the theoretical argumentation of the research as a means of providing evidence of its effectiveness.

The methodology of research consists in conducting students interviews, in the manner of Piaget and collaborators[5]. In the interviews the students are asked to search a word in a dictionary and are encouraged to explain in natural language how they did it and why they succeeded, as a first stage in the conceptualization. All the interviews were audio-taped and transcribed and in this case they have also been translated to English. The methodology of research has been inspired by APOS theory, a theory of learning mathematics elaborated by mathematics education researchers, based on Piaget's genetic epistemology[7, 1].

The paper is organized as follows: in the second section, the main ideas from Piaget theory are briefly described. The section also includes a summary of the identification of stages in the construction of the concept of the algorithm of binary search as a school subject. In the third section, the study is described including the design of the questions to the interviews, in the fourth section, the guidelines of the analysis of the responses of the students, selected excerpts from the interviews and the introduction of the formalization of the algorithm are included, and in the fifth section some conclusions and further work are presented. The sixth section includes acknowledgements.

2 Related works

The main ideas presented by Piaget and collaborators in works published in the sixties/seventies by the Series of "Etudes d'Epistemologie Genetique" (Presses Universitaires de France)[8–10] and in [11–13] constitute one of the sources of our investigation. Many of these works contain complete descriptions and analysis of several experiments, including selected parts of conducted interviews. In the experiments several types of problems are posed to determine the psychogenetic evolution of concepts, such as modus ponens, modus tollens and the transitivity of the implication.

The construction of these mathematical concepts follows the general law stated in "The Grasp of Consciousness"[8], according to which the process of construction of consciousness that accompanies the interaction between the subject and the object, always goes from the periphery (the result of the actions) towards the center (the intern mechanism of the actions). This general law explains why, at all levels, the conceptualization of the operations of the subject always occurs later than nonreflective use of the same operations.

On the other hand, in [13], the authors describe the instruments common to all acquisitions of knowledge and the processes that result from applying these instruments. The general source of the instruments are the processes of *assimilation* of representations of objects or events to the mental schemata of the individual, and the *accommodation* of the mental schemata in accordance with the representation to be assimilated. The instruments of knowledge generated by assimilation and accommodation are *abstractions and generalizations* in their diverse forms. The authors distinguish between the empirical abstraction, that draws information from the objects themselves, and the *reflective abstraction* that draws information from the actions and operations of the subject. As a result of abstraction, a reorganization into a new conceptual structure takes place, which is assimilated into the earlier one, allowing its generalization. The authors recall that *constructive generalization* is not only assimilation of new contents to already constituted forms, but generation of new forms and new contents [10].

And this is where the factor that plays a fundamental role as motor of the process comes into the scene: *the search for reasons and the inherent necessity.* The authors point out that a construction of the subject is not satisfactory, in the final analysis, unless it acquires intrinsic necessity, through explicit reasons [13].

The works of Piaget and collaborators have important pedagogical implications for the learning of mathematics and computer science concepts. The example presented in this paper illustrates the identification of three main stages in the construction of the concept of the algorithm of binary search as a school subject:

- An *instrumental* stage in which the students construct knowledge interacting with an instance of the problem without consciousness about how the solution is achieved and why it works. This knowledge constitutes the source of conceptual knowledge. For instance, the individuals solve the problem of searching a word in a dictionary (an instance of searching an element in an ordered list) but have difficulties in describing the steps and are unaware of the reasons of the success.
- A stage of *conceptualization* in which the evolution from the instrumental knowledge to conceptual knowledge starts by the grasp of consciousness and the reflection about the actions involved in the developed method and about the reasons of the success (or failure). This reflection leads to an understanding of the relationship between the structure of the elements over which the method is applied (the smaller parts of the dictionary) and the components of the method (choose, compare, search). This process ends in the comprehension of the algorithm and prepares the mind for the next stage.
- A stage of *formalization* which consists of constructing a correspondence between students' concept and a universal system of symbols. This is the stage that transforms a concept into a school subject. Students become aware of ambiguities and/or errors in their specifications and correct them gradually approaching a formal definition of the solution of the problem.

These stages act in a pro and retro-active manner, influencing the development of each other. For instance, the interaction between defining the method (formalization stage) and applying it to particular cases (instrumental-conceptualization stages) allows both to refine the definition and to improve the understanding of the algorithm.

3 The study

The study consists in conducting interviews to ten entering students of an introductory course of programming. The students are required to look up a word in a dictionary and to explain in natural language how they did it and why they succeeded. The problem is an instance of the general problem of searching an element in an ordered list and the solution is an instance of the algorithm of binary search. It is well known and automatically solved by the students. In the interviews they are encouraged to think about both the coordination of their actions (the method) and the modifications that these impose to the object (the structure). Accordingly to theoretical tenets, in this interaction the successfully done actions are transformed into operations leading to the conceptualization of the algorithm[8].

This task is adequate for the investigation because of the following characteristics:

- the algorithm of binary search is commonly applied in solving this task, that is to say, all students know very well the application of this algorithm to this particular case, and all of them success in searching a word,
- the task is (almost) automatically done, what gives us the opportunity of analyzing its grasp of consciousness and conceptualization in detail from the origin of the process,
- no numeric domain is involved and the role of school is minimal which diminishes the influence of preconceived ideas,
- this algorithm is one of the most important methods of searching and it is taught in all courses of programming, traditionally introducing some formalism to implement it, in the form of explanations given by the teacher to the students.

It is expected that the students solve the problem using the algorithm of binary search, without being aware of what that means. It is expected that the interviews help the students in understanding the algorithm with respect to:

- the structure of the dictionary as an ordered list of words,

– the actions that compose the algorithm: choosing a word, comparing words and a new instance of the search itself,
– the reasons of success: each new instance of the search is applied on "smaller" parts of the dictionary (do the same), all the smaller parts hold the property of containing the searched word (invariant), the search ends when a special case is achieved (termination).

Accordingly, the series of questions of the interviews is divided into the following segments (Q1 means question 1, etc):

– Q1 to Q4: Structure
– Q5 to Q6: Method (at the beginning of the interview)
– Q7 to Q8: Reasons
– Q9: Method (at the end of the interview)

3.1 Designing the interviews

The enumerated questions listed below were used as a basis for the interviews. In some cases, other questions were added at the moment or the same question was formulated in another way, depending on the development of each interview. The whole content of the interviews can be found in [3]. The Spanish word for "cat", that is, *gato* is maintained as a remainder of the original language of the interviews.

Questions 1 to 4: The first four questions are aimed, on one hand, to establish a fluid contact between the student and the interviewer and on the other hand, to induce the student to reflect about the structure of the dictionary as a list of words alphabetically ordered. It is the existence of this order relationship that determines the method of searching and consequently, we think that it is a relevant concept that has to be assimilated by the minds of the students as a requisite for understanding the method and the reasons of success.

Q1: What is a dictionary?
Q2: Knowing that a word, for example *gato*, is in the dictionary and also in this novel, where do you think that will it be simpler (easily and quickly) to find it?
Q3: Why?
Q4: What makes the difference then between a dictionary and any other book?

Questions 5 and 6: The goal of the following questions is to apply Piaget's ideas about the conceptualization of automatically done actions. Interrupting the action and introducing the need for thinking forces the student to direct his/her thought from the result of the action towards the intern mechanism of the coordination of the actions that has given rise to that result. This movement "from the periphery to the center", that Piaget calls *The general law of cognition* [8], allows the subject to construct better representations, on the one hand, of the objects and on the other hand, of his/her own actions which are transformed into operations (methods). In this case, the conceptualization of the method demands to interrupt the automatic application and to mentally identify the sequence of other involved methods: to choose a word, to compare it with the searched word, to make a decision according to the result of the comparison, to do the same (another instances of application of the same method).

Q5: Look up the word *gato* in the dictionary.
Q6: Describe, step by step, how you achieved it. (Eventually, ask them to do it again).

Questions 7 and 8: The goal of the following questions is to apply Piaget's ideas about the role of the process of "searching the reasons of success" in the conceptualization. The constant motor impulsing the subject to complete or to replace the observables of facts by deductive or operative inferences is the search of *reasons* for the obtained result [10, 11]. That means that "the search of reasons" impulses students' thought towards the interaction between the coordination of his/her actions and the modifications imposed to the object, reaching an equilibrium generating a mental representation of the algorithm. That means, for the case of this algorithm, make the students aware of the relationship between the arguments to which different instances of the method are applied: each new instance of the search is done on "smaller" parts of the dictionary (all holding the searched word), until a special case is achieved (termination).

Q7: Knowing that a word is in the dictionary, is it always possible to find it?
Q8: Why?

Question 9: This question is essentially the same as Q6 but is posed at the end of the interview. By comparing the responses to both questions (Q6 and Q9), the impact of the interview on the levels of conceptualization of the algorithm of binary search can be appreciated. The goal of the question is to determine whether the interview has helped the students to improve their levels of conceptualizations with respect to, on one hand, the decomposition of the algorithm in its components and on the other hand, the description of a general algorithm. The epistemological motivation arises from the fact that the ability of detaching the thought from particular cases and comprehending the intern mechanism of the actions is considered an advance in the conceptualization by Piaget.

Q9: If you had to explain to a little child -who knows how to read and knows the alphabet- how to look up any word in a dictionary, what would you say?

4 Analysis of the information gathered in the interviews

As expected the students solve the problem using the algorithm of binary search, without being aware of what that means. Through the interviews, the students transform this "know how to" into knowledge about "what is done" and "why it works", by the process of reflective abstraction [12].

The analysis of the information gathered in the interviews is organized in the following segments:

- Responses to questions on structure and method (Q1 to Q6).
- Responses to questions on reasons of success (Q7 and Q8), including excerpts from the interviews.
- Responses to the question on the method posed at the end of the interview (Q9), including responses to Q6, to facilitate the observation.

The analysis of every segment is based on descriptions and explanations contained in the respective references. Selected excerpts from the interviews are included, in which relevant comments relating theoretical concepts to the responses of the students are indicated in italics. A short description of introducing the formalization of the algorithm using as start point the conceptual knowledge constructed by the students is included following the analysis.

4.1 Responses to questions on structure and method (Q1 to Q6)

The responses to questions Q1 to Q4, constitute examples of how the thought advances from the periphery (inferences focused on the characteristic of the dictionary related to its use) towards the center of the object (inferences focused on the structure of the dictionary), induced by the corresponding questions. This movement is governed by tenets called by Piaget *the general law of cognition* and is the start of the process of conceptualization [8].

Only one student characterizes the dictionary by its alphabetic order relation. The other students refer to what one can do with the dictionary or what it is for, in other words, their responses are adequate to questions like "what can you do with the dictionary?" or "what is the dictionary for?". Because of the obvious character of the property of being an ordered list, it is more difficult to conceptualize. Faced to questions Q3 and Q4 (why is it easier to find a word in a dictionary than in a novel, inducing to think about the difference between both structures), all students recognize the alphabetical order, which reveals that they become aware of the relevant property of the object.

The answers to question Q6 reveal that the students have a very weak conceptualization about their method of searching a word. The students apply binary search as expected, but they are not aware of the different actions they do to achieve the result. The questions following Q6 (Q7 and Q8) play an essential role in the conceptualization of the method, inducing the students to reflect about the reasons of success. The goal is that they achieve on the one hand, the decomposition of the method in its components: *choose a word, compare words* and *do the same*, and on the other hand, the transformation of the dictionary in smaller parts, which are the central questions of the algorithm. To facilitate the observation of advances in the conceptualization, some responses to question Q6 are included together with responses to question Q9.

4.2 Responses to questions on reasons of success (Q7 and Q8)

The analysis of the responses of the students to questions Q7 and Q8 revealed that it is hard for the students to understand the reasons by which the binary search works. New questions (no numbered in the excerpts below) were added in order to help the students to surmount this difficulty: they were asked to use another method (called "the second method" in the following) which consists in asking the students to open the dictionary and to look if the searched word is there. If it is not, (which will happen in essentially), they are asked to close the dictionary and to begin to search for the word again, that is, in the whole dictionary. This strategy was effective: after using the second method, all the students immediately became aware of the changes that their actions impose to the object: the sequence of parts of the dictionary, *each time smaller* all of them holding the searched word, and the termination case.

Below, selected excerpts from the interviews illustrate about:

- the difficulties of the students in finding the reasons of success before using the second method (student 1),
- the effectiveness of the strategy of using the second method (students 2, 3 and 4),
- advances in the conceptualization of the algorithm with respect to: the decomposition of the method in its components (choose, compare, search) (student 2), the invariant property (students 3 and 4), the termination case (students 1 and 3).

The responses of the students to questions Q7 and Q8 are classified on the following types, accordingly to the factor that they point out in searching the reasons of success:

- type 1: both the method and the dictionary,
- type 2: the method,
- type 3: the dictionary.

The goal of the questions is to induce the movement of the students' thought between types 2 and 3 above, accordingly to the general law of cognition [8].

Selected excerpts from the interviews

Student 1
Q7: Knowing that a word is in the dictionary, is it always possible to find it?
R: Yes.
Q8: Why?
R: Because it will always be in that order. Because looking for in this way and due to the order the dictionary has, I will find it.
(He refers to the method and the dictionary: type 1)
Q: Good, we look for the word *gato*. Here, we are searching here, in this piece as you said and, what is it like regarding the whole dictionary?
R: The rest.
Q: And what is this rest according to the whole dictionary?
R: More useful for me.
Q: Yes, and with respect to the number of words that has?
R: Greater.
Q: ... !! This part is with respect to the whole dictionary?
R: Smaller than the complete dictionary.
Q: And now, how do you go on searching?
R: (He does it)
Q: And now what happens with all this part?
R: I do not need it.
Q: And where will you search?
R: Only on this page.
Q: And what is this page like according to the previous section you had?
R: Much smaller.
Q: Then what happens with the dictionary when you are still searching?
R: Some parts are being discarded.
Q: And it becomes ...?
R: Smaller.
Q: When you find the word, what happened to the dictionary?
R: It is useless, it only helps me for that word only.
Q: So, what was the dictionary transformed into?
R: Into only one word. *(Termination case.)*
Q: That is why you find it, because you search in sections that become smaller and smaller within the dictionary.
R: Oh, of course.

The first type of response (illustrated in this excerpt) reveals at first sight a better conceptualization of the reasons of success because both the method and the object are mentioned. However, observe the difficulty of the student, in constructing a mental representation of the relationship between the parts of the dictionary. This can be explained by the fact that this student was interviewed before the strategy of using the second method was implemented.

Student 2
Q7: Knowing that a word is in the dictionary, is it always possible to find it?
R: Yes.
Q8: Why?
R: Because I always get the same method to look it up, I always apply the same method by which ... everybody uses it for the dictionary.

(*He refers just to the method: type 2*)
Q: And that method you apply and that everybody uses, why does it guarantee you that you will find it?
R: If it is in the dictionary?
Q: Yes.
R: ...
Q: What does this method have that guarantees you find it?
R: It never fails.
Q: Why?
R: ... because ... because it holds all letters in the dictionary, I mean I always ... never will get lost ... as long as I search one ... then I will always follow the same order until I find the word, by each letter I go on searching.
Q: I propose another method. (He does it using the second method).
R: Yes, of course, we never finish.
Q: Which is then the difference between this method and the other?
R: I based myself on the principles (*He is thinking of his own action now, trying to decompose it*), that is, I open the dictionary and I based myself on what I find at first sight, (*choose*) then I start checking if I have to go forwards or backwards (*compare*).
Q: Suppose you go forwards, what happens to the rest?
R: I discard it.
(*Observe that he is still thinking of his own action. However, to advance in conceptualizing his thought has to move to the transformation imposed to the object because of his action. That is what next question induces.*)
Q: Then, what happens with the dictionary while you are looking up?
R: I start discarding until I find the word I am looking for.
Q: Then, what happens to the dictionary while you are searching?
R: They are being eliminated, it becomes reduced (*"smaller" arguments*).

Student 3
Q7: Knowing that a word is in the dictionary, is it always possible to find it?
R: Yes.
Q8: Why?
R: Because I trust myself, because yes, (she laughs).
Q: You'll find it rather quickly or you will take the whole day.
R: Quick.
Q: Why?
R: Because I have practice. *(Observe that she is thinking of her own actions, response of type 2.)*
Q: Now we are going to use another method (the second method). Open the dictionary (she opens it).
Q:Is *gato* there?
R: No. (We do it again many times).
Q: What do you think of this method with respect to the other?
R: It's much more difficult.
Q: And what is the difference between them?
R: That in the first one I start marking between which and which and I can shorten the limits (*"smaller" arguments*) and in the other it is at random.
Q: Why are you sure that with your method you will find it and quickly?
R: I have fewer bounds, fewer limits. (*She begins to think of the dictionary*).
Q: What happens with the group of words in which you search?
R: They start shortening.
Q: Until?

R: There is one word, the searched one (*Termination case*).
Q: If it is not?
R: It is not possible, I choose the parts in which the word is ... (*The selected part contains the searched word*).

Student 4
Q7: Knowing that a word is in the dictionary, is it always possible to find it?
R: Knowing that it is in the dictionary, yes.
Q8: Why?
R: Due to the order it has.
(*Response of type 3 above, he attributes the success to the dictionary.*)
Q9: Good, let's use now another method to find the word *gato* with the same dictionary having the same order. We close the dictionary and I ask you to look up the word *gato*.
R: (He does it).
Q: Is the word *gato* there? (Where he opened).
R: No.
Q: Close it and look up *gato* again.
R: (He does it).
Q: Is it there?
R: No. (It is repeated some times).
Q: With this method the dictionary keeps its order and however, do you think that in this way we'll find the word *gato* easily?
R: No.
Q: Instead, with your method?
R: I find it fast.
Q: What is the difference between the two methods?
R: One is safe and the other isn't.
Q: Why?
R: Because there is an order.
Q: No, in the second method the order remains in the dictionary.
R: But I didn't follow it.
(*Observe that now his thought has moved to his actions.*)
Q: What does "following the order" mean? Let's use your method again.
R: (He does it.)
Q: Why are you browsing in that direction?
R: Because there is letter "e" and I know that letter "g" is after.
Q: What happens then with all this part of the dictionary?
R: I discard it. I know that here "g" is not and I discard it.
Q: Let's go on.
R: (Goes on searching and he passes by).
Q: What do you do with all this part of the dictionary?
R: Also, I know that it is not in this part.
Q: What happens with the dictionary?
R: There are parts which I know that it is not there and I discard them. (*The selected part contains the searched word.*)
Q: So what happens to the dictionary?
R: It becomes smaller (*"smaller" arguments*).

4.3 Responses to question Q6 and Q9

The responses to questions Q6 and Q9 allow to some extent to measure the impact of the interview in students' conceptualization of the algorithm, because they are essentially the same question, posed at the beginning and at the end of the interview respectively. Evidence of the advances are the references to a general algorithm and better descriptions with respect to the decomposition of the method in its components actions: *choose a word, compare words* and *do the same*. Selected responses to the questions Q6 and Q9 follow.

Student 1:

Q6: I check the beginning letter, with "g", there is an alphabetical order then, more or less I know that it is in the middle upwards, then I check which letter I am, for example: If I open to the "e", I know that "g" is after, that is the order, I go forwards, I go on checking and if I advance too much, I come backwards and so on, *until I find "g" and after that I search.*

Q9: First that he opens the dictionary, that he reads one word (*choose*) and he defines the relation, that is what he tells me if the letter with which he started is before or after than the one he wants to find and he tells me that if it is after (he passes by) I ask him to choose another that is before (*compare*) and in this way successively (*do the same*).

Student 2:

Q6: First I opened the dictionary in a section that seemed near to "g" and went in order always visualizing the alphabet in my mind. I went up to "g" and then with the 2nd letter, which is the "a", and then the "t" *until finding the word.*

Q9: That the word that he wants to find will be in order according to the alphabet that he knows. Then, what he has to search first is the beginning letter of the word. After having found the first letter, he can start with the 2nd. Proceeding also in the same way.
Q: The kid is learning to search ... Then, when he opens the dictionary, what does he have to do to find the 1st letter?
R: A relation between what he is seeing (*choose*) and what he wants to find and according to this relation (*compare*) he "operates" forwards or backwards.
Q: And when he finds the first letter?
R: He operates again using the same order (*do the same*).

Student 3:

Q6: I was checking on the letter according to the order that I have in my mind, *first, the first letter, secondly the second and so on.*
Q: Doing what?
R: Discarding the ones didn't help, and looking up the words with the letters I was searching for.

Q9: Take the dictionary and choose one word (*choose*) and then with respect to that one, start looking if it is greater or less (*compare*) until finding the 1st letter and after you found the 1st letter, you have to look for the 2nd , in order as well, and go on until you find the whole word (*do the same, termination*).

In the answers to question Q6 the actions involved in the method, are implicitly mentioned for the case of the letters of the word, while in the answers to question Q9 most of students explicitly talk about "a relation between words" (*choose, compare*), "proceeding in the same way" (*do the same*), and in some cases also about the termination condition. On the other hand, responding to question Q9 most of students describe a general algorithm of binary search. The next subsection shows the introduction of a formalization of the algorithm derived from a synthesis of students' responses to the question Q9.

4.4 The stage of the formalization

A week after the end of the individual interviews, a collective class was taught to all students to represent students' descriptions in a formalism similar to mathematics. The meaning of formalizing is to put into correspondence mental constructions (concepts) with some universal system of symbols [9]. Traditionally, in computer science teaching algorithms are described using different formalisms (pseudo-codes, diagrams, flow-charts). But these formalisms are not universal systems of symbols, while mathematics is. The following notation is used to describe students descriptions in a language similar to mathematics: the dictionary is represented by a list of pairs (word, meaning). A definition of the method of searching a word in a dictionary is written as:

```
search(word,[(Wfirst,meaning)...(Wlast,meaning)]) =
      let word' = choose-word([(Wfirst,meaning)...(Wlast,meaning)])
      if word = word' then select-the-meaning
      else if word'<word then search(word,[(word',meaning)...(Wlast,meaning)])
             else search(word,[(Wfirst,meaning) ... (word',meaning)])
```

Each student was provided with a sheet containing his/her responses and once the students have worked this definition out, other questions are discussed, for instance, "How can we define a more efficient method 'choose-word' in the searching algorithm?" The term efficient is not specified to see what the students think about it. All the students have answered that they should take the word from the middle, and that this is more efficient because there is more possibilities of discarding a greater number of words.

At the end of the class, some exercises were handed out to the students to be solved individually or in groups. The exercises asked about problems presenting both similarities and variations with respect to the solved problem, for instance,"What happens if the word is not in the dictionary?" and "How would you do to insert it in the dictionary?" Such questions give the opportunity of comparing different solutions/problems and reasoning about efficiency issues in a context that the students can easily understand.

The analysis of students' responses to those questions allows to investigate how the constructed knowledge is used in solving new problems[10] (not included in this paper).

5 Conclusions and Further Work

The study described in this paper is an example of the contribution of Piaget's theory-*Genetic Epistemology*-to computer science education research. The paper shows how principles from the theory can be applied to learn about students' understanding, using as an example the binary search algorithm. The design of the interviews (section 3) and the analysis of students' responses (section 4) follow the main ideas briefly described in section 2.

The selected excerpts from the interviews reveal the impact on the construction of the concept of binary search algorithm of

- *the process of interaction* of students' thought between the applied method *and* the modifications on the object, and
- students' reflection about *the reasons of success* of their solutions to the problem.

Advances on the conceptualization are illustrated by students' descriptions at the end of the interviews. A synthesis of these descriptions is used to introduce a formalization of the algorithm in a language similar to mathematics, as described at the end of section 4.

The future work focuses on describing parts of the research not included in this paper, for instance, the implementation of the algorithm in the functional programming language

Haskell[1] and the investigation of applying the constructed knowledge to solve other problems, as mentioned at the end of section 4.

To discuss ideas related with the study described in this paper, a series of workshops were conducted during the second semester of 2007 with the participation of computer science educators at Instituto Universitario Autónomo del Sur[2]. In the last years, some innovative methodologies taking into account the activities developed in the workshops have been put in practice[6]. The analysis of these experiences is also one of the points of the further work.

The elaboration of instructional proposals about the learning of algorithms, aimed at providing computer science educators with teaching strategies, is a matter of the further work as well. Many algorithms used in real life to solve problems or perform tasks are instances of general algorithms, taught in programming courses, for instance, searching (binary and sequential), inserting/deleting elements in lists, ordering lists. On the other hand, the students learn how to use algorithms in mathematics courses that can serve the same purpose[4].

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