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‘It’s just like the whole picture, but smaller’: Expressions of gradualism, self-similarity, and other pre-conceptions while classifying recursive phenomena

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Abstract

Recursion is a key concept, appearing in almost every introductory course in computer-science. Educators and researchers often refer to difficulties in learning and teaching recursion. However, the research literature barely addresses the unique ways in which students relate to this interdisciplinary concept and the particular learners’ language concerning recursive phenomena. The gap is most apparent when seen through a constructivist lens, where the students’ prior knowledge and idiosyncratic conceptions are referred to and reflected upon in order to serve as a basis for further knowledge construction. In our study, high school students collaboratively classified several recursive phenomena and discussed their criteria and categories with each other. This paper focuses on a part of the study that deals with a variety of pre-conceptions which emerged from analysing the students’ discourse, and suggests a model for organizing these pre-conceptions. Our findings could contribute to the recognition of the role of class discourse in the process of constructing the concept of recursion in particular, and in learning abstract computer science concepts in general.

Introduction

The utterance in the title ‘it’s just like the whole picture but smaller’ was expressed by Pavel, a sixteen years old Israeli student. Pavel was trying to draw the attention of his classmates to a certain characteristic of a recursive phenomenon he had just recognized, while looking at and analyzing the tree in Figure 1.

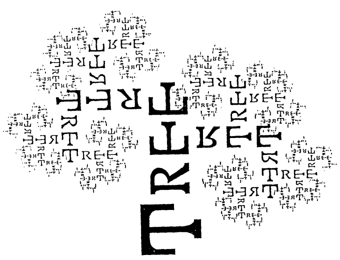

Figure 1 – One example of a recursive phenomenon (Unknown source)

Pavel was participating in a functional programming course as part of his computer science curriculum in high school, and his class was just at the beginning of the major part of the course, which dealt with recursion. The students were first exposed to recursion while participating in a constructivist and interdisciplinary learning activity, during which the students classified many different examples of recursive phenomena according to some criteria of their own choosing, and agreed upon names for each class of examples.

The entire activity divided into four different learning phases. In Phase 1, the class was first exposed to recursive phenomena, and in Phase 2, the students worked in groups to classify the phenomena. In Phase 3 each group presented its classifications, categories and criteria. In Phase 4 the teacher guided a reflective discussion with the whole class (for more details on the so-called Classification and Discussion Activity - CDA, see Levy & Lapidot 2000).

Among its other advantages, such a learning activity can stimulate a very rich class discourse concerning both the specific examples of recursive phenomena and the general concept of recursion, as the learners express their unique way of conceiving, discussing, arguing, thinking and understanding. During one such lively class discourse, Pavel was documented as he expressed his pre-conceptions of self-similarity and gradualism by saying 'it's just like the whole picture but smaller'.

The above was chosen among many such utterances, statements and non-verbal expressions that were documented and gathered during the course of a research project focusing on high school students' discourse of recursive phenomena. Throughout the inductive analysis of the discourse, the students' expressions were interpreted, refined and formulated as pre-conceptions. This paper presents these pre-conceptions, suggests an organizing model, and discusses some implications of the research findings.

Research on learning recursion in high school

Recursion is a core concept in computer science, and many agree that, while it is powerful and significant, it is difficult to learn and understand (see George 2000 for a comprehensive list of general references, and Leron 1988 for a specific suggestion as to the origins of this difficulty). Issues of learning recursion sometimes appear in textbooks accompanied by a warning that it is not an easy concept. It has been observed that "teaching students to use recursion has always been a difficult task. When it is first presented, students often react with a certain suspicion to the entire idea, as if they had just been exposed to some conjurer's trick…" (Roberts 1986 p. vii. See also Wu, Dale & Bethel 1998).

There seems to be an overall consensus on the difficulty of teaching recursion to novices studying computer science, both at the college and high school levels, yet the literature hardly refers to the latter (Ben-Ari 1997 is one exception). Moreover, educational research has not emphasized the learners' voices or the learning processes, as they are manifested in a natural setting, in a real class dealing with computer science concepts.

Special importance is given to recursion within the functional paradigm, yet the issue of difficulties in learning recursion in functional programming environments has also hardly been addressed. "It turns out that the idea of recursion is both very powerful - we can solve a lot of problems using it - and rather tricky to understand", state Harvey and Wright when introducing recursion within a functional programming environment (Harvey & Wright 1993).

Following his rich experience in teaching recursion via Scheme and Logo, Harvey suggests several different ways to explain recursion. Teachers and curriculum developers can use Harvey's methods, together with other teaching methods the literature suggests. Still, those who wish to understand the unique conceptions of novice high school learners remain dissatisfied, and questions regarding how computer-science classes cope with learning recursion remain unanswered.

We can further ask how one can characterize the class discourse and how this discourse reveals learners' conceptions. All these were included in the questions that were addressed in our research on the process of learning recursion. This paper presents our primary answers.

The next section briefly describes some methodology considerations. The results section that follows may help bridge the gap described above, by exposing the wide range of pre-conceptions high school learners express as they undertake to learn recursion. In the last section we conclude with the recognition of the role of class discourse in the process of learning recursion.

Methods for documenting and analysing the class discourse

The focus of the research presented here was on the first phase of the recursion learning process, as it occurred in eleventh grade classes. These classes had just begun the intermediate period of their functional programming course (see Lapidot, Levy & Paz 1999 for details on the course). At that point, the learners were first exposed to the concept of recursion, by participating in the four learning phases of the classification and discussion activity.

Our research goal was to document and analyse learners' discourse on recursive phenomena throughout the entire learning activity, as a way to look at recursion through the learners' eyes and to understand their unique ways of conceptualising and articulating the general idea of recursion.

The class discourse was recorded and documented using observational field notes in six different classes. The recordings were then fully transcribed, and the transcriptions, together with the field notes, served as the source for an inductive discourse analysis. According to this method of analysis, "As you read through your data, certain words, phrases, patterns of behaviour, subjects' ways of thinking, and events repeat and stand out… These words and phrases are coding categories" (Bogdan & Biklen 1998 p. 171).

In our first phase of analysis, three analytic perspectives, or dimensions, were identified in the students' discourse: the content, the cognitive, and the communicative dimensions. Starting from the content perspective, we then concentrated on what the students discussed, and used their words, phrases, drawings and written works as coding categories. The next section presents these emerging content categories, and interprets them as pre-conceptions. The other two analytic perspectives that our study took into account will be described in a future publication.

Results: pre-conceptions expressed by high school students

Before listing the key preconceptions that emerged in the course of the discourse analysis, we will present one class episode that was documented at Phase 4, at the end of the learning activity (see Levy & Lapidot 2000 for sample episodes from previous phases). In this episode, the students expressed their ideas concerning the common features of the recursive phenomena they had just classified. In other words, the students attempted to articulate and generalize the *sameness*, instead of their previous efforts to articulate the *differences*. Table 1 contains both the discourse and our interpretation of the events.

The discourse in Table 1 hints at the students' making use of cognitive acts such as naming, comparing, classifying and generalizing (Feuerstein, Rand, Hoffman, & Miller 1980), but as mentioned before, this analytic perspective will not be dealt with here. What we do focus on are those grey-highlighted words and phrases appearing in Table 1, or the content expressed by each episode. These expressions may provide insight into the students' unique ways of thinking about the recursive phenomena they had been investigating.

Documented Discourse	Interpretation
Student1: Dependency…In the hands, if one doesn't draw the first, then… Student2: Here we can't see the end, but there is an end anyway. It clashes, these leaves will clash. They must clash. Student3: Theoretically you can go on. Student4: It can go on, but you won't see it. Student5: It is repeating. Periodic. (In Hebrew: 'Chozer. Machzori').	Student1 is talking about Escher's famous image of two hands drawing each other. Student2 and the others are talking about Figure 1, trying to articulate what would happen if you tried to draw more and more levels of the tree.
Student8: The aim of all this is to fit inside… A function that fits inside another a function, a function that calls a function. For example, the function 'Ronen' takes the function 'Jonathan' and adds one, but the function 'Jonathan' takes the function … say, 'Alfonso'… That's the idea; a thing is built upon a thing.	Student 8 refers to all the recursive phenomena he has recently encountered. He uses his prior knowledge of functional programming, especially the knowledge of combining functions.
Student5: There are also loops and infinity here. We can combine the functions so that the combination would be endless. Student7: Here it stops! Student5: Everything here is periodic…	Student5 describes the whole range of recursive phenomena. Student7 refers to the recursive definition of factorial .

Table 1 – A sample class episode (case 14, December 1999)

When Student5 said ‘everything here is periodic’, she expressed her own way of perceiving and characterizing the various recursive phenomena she dealt with, using what we as researchers call the *periodic* pre-conception (‘Machzori’ in Hebrew). When Student8 said ‘the aim of all this is to fit inside’, he expressed what other students expressed when they talked about *containing* (‘Hachala’ in Hebrew). When Pavel, the student quoted in the title of this paper, said ‘it’s just like the whole picture but smaller’, he expressed both the pre-conception of *gradualism* and the pre-conception of *likeness* that we later reformulated as *self-similarity*. These unique, student-formulated phrases were part of the large amount of data we gathered, analysed, and finally classified as pre-conceptions representing the students’ ways of thinking about recursion. The label ‘pre-conceptions’ was chosen considering both the conceptual nature of the discourse, and the preliminary phase of the learning process in which the students had been involved.

Analysing the discourse, we came up with a diverse collection of two dozen different content categories, where each category includes expressions that suggest a similar way of talking about recursive phenomena. Table 2 presents a notable third of the content categories, namely eight categories that are considered as key pre-conceptions. These key pre-conceptions appeared most often in the students’ discourse and were remarkably associated with other pre-conceptions.

Each key pre-conception in Table 2 is illustrated by an utterance expressing it. These representative utterances were selected among the data gathered at six different classes (titled as Case 9 … Case 14). In order to provide an expanded view, the right column of the table presents the various other pre-conceptions that tended to be associated with each key pre-conception.

The key pre-conception	**Example of an utterance expressing the pre-conception**	**Other associated pre-conceptions**
a. Infinite or Finite (I/F)	“It stops the whole process” (Case 9)	Returning,Sequential, Gradualism, Circular, Periodical, Repetition
b. Regularity	“There is a kind of a rule here” (Case 10)	Gradualism, Withdrawal, Periodic, Sequential
c. Gradualism	“From the big one to the little one and vice versa” (Case 10)	I/F , Regularity, Periodic, Sequential, Split, Withdrawal
d. Periodic ‘Machzori’	“It is repeating. Periodic” (Case 14)	Circular, Gradualism, Regularity, Repetition,Sequential
e. Returning ‘Chozrim Bachazara’	“Here it returns to beginning” (Case 9)	Reflection, I/F, Dependency
f. Sequential	“There are increasing and decreasing sequences here” (Case 10)	I/F, Gradualism, Regularity, Periodic
g. Dependency	“There is a kind of … dependency on the former” (Case 14)	Withdrawal, Sequential, Mutuality
h. Self reference	“Things that build themselves” (Case 11)	I/F , Circular, Fractal, Containing

Table 2 – Key pre-conceptions expressed by high school student

So far, we have briefly described the collection of pre-conceptions that we could distinguish through the discourse analysis. Two important findings can be summarized here:

- First, high school students indeed expressed a rich and complex conceptual scheme when they were first exposed to the idea of recursion. Within the framework of social constructivism (Confrey 1995), such findings may indicate that social interaction can stimulate elaboration of conceptual knowledge.

- Second, some key pre-conceptions were highly linked to others, while other pre-conceptions tended to be more isolated. Moreover, two integrated pre-conceptions were recognized in the discourse. The first '*Regular gradual recurrence*' combined the key pre-conceptions b, c, and d. The second '*Infinite gradual recurrence'* combined a, c, and d. It is significant that the students' discourse not only hinted at the components of their conceptual schemes, but also hinted at the process of reconstructing these schemes by expressing linkages and relations (Hiebert & Lefevre 1986).

As a further analytic step, we also looked at the four different phases of the learning activity through which the class discourse was documented, and organized the pre-conceptions according to the phases in which they appeared at. Our suggested model for organizing the pre-conceptions is presented in Table 3, where the phases that were found relevant for each pre-conception are highlighted.

The pre-conception	**Phase 1**	**Phase 2**	**Phase 3**	**Phase 4**
Returning				
Infinite or Finite	1	2		
Circularity				
Containing				
Split				
Reflection				
Symmetry				
Sophistry				
Self-reference				
Self-similarity				
Regularity				
Regular gradual recurrence				
Gradualism				
Periodic				
Sequential				
Withdrawal				
Infinite gradual recurrence			3	
Dependency				
Fractal				
Mutuality				
Function that calls itself				

Table 3 – A model for organizing the pre-conceptions

The model above includes most of the pre-conceptions that were identified. The different outlined boxes that define three sections of the model (numbered 1,2,3), suggest three additional findings:

- Some pre-conceptions appeared as early as Phase 1, and continued to be expressed throughout the learning activity. In our research, the pre-conceptions which appeared most consistently throughout the phases were *Infinite or Finite*, *Circularity* and *Containing*.
- The group discussion phases (Phase 2, Phase 3) proved to motivate a rich expression of pre-conceptions. Many of the various pre-conceptions were rooted in these learning-without-guidance phases. Following Krummheuer (1995) and others, we suggest that the argumentative nature of the group discussions might be responsible for that richness.
- There seems to be a terminological shift towards and throughout the last, reflective Phase 4. During that phase, the students used a slightly more formal language, e.g. their use of

Symmetry, *Dependency*, *Fractal* and *Mutuality*. The lingual change might also reflect a conceptual change, by expressing the process of collaborative reconstruction of ideas and by pointing to the communal dimension of learning (Confrey 1995, Cobb 1996).

We classified these findings as the consistency of pre-conceptions; the cognitive potential of group classification and discussion; and the creation and refinement of a class genre appropriate for discussing the idea of recursion (for a discussion of class genres, see Karasavidis, Pieters & Plomp 2000). Together with the findings discussed earlier, namely the diversity of the pre-conceptions that high school students expressed and the conceptual network that they weaved, five different results have been discussed in this paper. We believe that such discussions can illuminate the processes by which learners construct an abstract concept like recursion, and can draw an interesting and unique picture concerning the ways in which students relate to this interdisciplinary concept and the particular learners' language concerning recursive phenomena.

Implications for understanding high school students' conceptions

The implications of our findings could be significant both for understanding how students construct the conceptual scheme of recursion, and in providing a more general understanding of construction and reconstruction processes. Out of considerations of space, we would like to point out only one implication, which is well documented by researchers in the discipline of mathematics education.

The pre-conceptions that emerged in our research hint at the interesting distinction between the more operational kind of conception and the more structural kind of conception. This issue has been raised by our own discourse analysis and by contemporary theories of mathematics education. Following Piaget, some researchers offered to look at the process of constructing abstract mathematical concepts as a gradual process, in which the learner moves from an operational conception towards the more developed structural conception (Sfard & Linchevski 1994, Breidenbach, Dubinsky, Hawks & Nichols 1992). When expressing an operational conception, the learners focus on actions and processes, as could be the case for our students that expressed pre-conceptions like *Infinite or Finite*, *Gradualism*, and *Periodic*. On the other hand, focusing and expressing the pre-conceptions of *Containing*, *Fractal*, and *Self-reference* could be interpreted as representing a more structural conception of recursion.

It was interesting to discover that all the different kinds of conceptions were present in the same class. Moreover, they often existed harmoniously within a single utterance expressed by the same student, as is the case in Pavel's utterance in this paper's title. Such harmony contradicts former findings concerning the superiority of the operational conception of recursion, even when the students expressing that kind of conception were not novices (Aharoni 1999). The operational conception of recursion might be a consequence of a programming-oriented thinking, constructed by over-emphasizing computing and algorithmic aspects of recursion throughout a programming-oriented curriculum. The implication for teaching computer science concepts in general, and for teaching recursion in particular, could be that curricular steps should be taken in order to emphasize the structural aspects of concepts like function, variable, and recursion.

In an even more general sense, we would like to conclude this paper by emphasizing the important role of class discourse in the process of constructing recursion and other scientific concepts (Mortimer & Machado 2000). Such recognition could help educators and researchers in understanding students conceptual scheme, in helping students reflect upon and reconstruct their conceptions, and in improving teaching and learning. Within the young and growing research community of computer science education, such recognition is crucial.

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